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The Reconstruction of the Quartic Bezier Curve Using Uniform Parameterisation

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Abstract: This study proposes the reconstruction of a higher Bezier curve using uniform parameterisation to improve the design of a curve. The parameterisation process is the key element in the reconstruction of the curve. The reconstruction process starts by selecting an image. Next, the boundary is extracted from the image. Using uniform parameterisation, the parameter value of the image is determined. Then, new control points for the image are calculated. In the next stage, the construction of the image is implemented. The finding displays an improved reconstruction compared to the initial curve. The reconstruction proposed in this study is provided with systematic and scientific techniques that can benefit designers, businesses, and the craft industry. This proposed study can also enhance the creativity and innovation of the design industry.

Keywords: Bezier curve, Control points, Uniform parameterisation, Reconstruction,

1 Introduction

Consumer spending in creative and design industries contributes to economic growth. However, the evolution of the creative industries' craft area is not optimal due to a lack of aesthetic innovation and attractiveness. To boost attractiveness, a variety of relief objects is required [3]. Companies, such as in the batik industry, should apply computational designing and printing as an alternative in their operations.

Several aspects of the industries' crafts need to be improved, such as the product design. If we look at today's products, for example, there is still an error in flattening the curvature, particularly around the lip of the bottle. Higher consumer demand can lead to various designs and colours, such as in the batik industry. The design possibilities are endless with time-saving computer designs.

The most crucial requirement for designing curves is managing the curvature and smoothness of the curve. In Computer Aided Geometric Design (CAGD) and Computer Graphics, Bezier curves and surfaces provide a variety of essential tools for modelling [4]. Bezier curves were created by French engineer Pierre Bezier in 1962 to design car bodies [2]. A Bezier curve is constructed from the control points and is generally used in computer-aided geometric design (CAGD) [1]. The points can be exhibited visually and used to adjust the curve intuitively because it is entirely contained within the convex hull of its control points. Consequently, this study proposes the reconstruction of a higher degree curve, a quartic Bezier curve, using uniform parameterisation to improve the reconstruction and smoothness of the curve.

2 Literature Review

A study by [3] proposed a mathematical model of the quartic curve Bezier from the modification cubic of the Bezier curve. Based on the object design, a combination of both geometries could be used. The Bezier curve is a polynomial-based curve that is often used to create items and form their surfaces. According to [3], the development of the higher degree Bezier curve is needed, as is the addition of a curve modification to create a variation of curve curvature that can be used to simulate industrial products such as glassware. The study from [3] involves designing a Hermite quartic matrix, finding new control points, constructing the equation of the Bezier quartic from the modification of cubic Bezier, and developing a quartic rotational surface shape from the modification of cubic Bezier. The importance of this approach is that a mathematical understanding of the Bezier curve can enhance the design potential in creative industries.

Moreover, a study by [8] investigated the properties of the complex quadratic Bezier curve on the unit circle and the connection between the complex quadratic Bezier curve and the Pascal spiral curve. Furthermore, their study highlighted that the curve was contained within the convex body of its control point, which might be represented and intuitively used to alter the curve. Curves can be applied to affine transformations like translation and rotation by using their respective transformations at the curve’s control point. To ensure smoothness, the control point, where two curves met, must be on the line between the two control points on either side. Therefore, to understand the unique inflection and convexity of the complex quadratic Bezier curve on the unit circle, Li and Xue [8] stated that one must first understand the characteristics of all complex quadratic curves on the unit circle.

Wang et al. [9] proposed a geometric model for Bezier curves with monotone curvature and introduced a new method for constructing monotone curvature (MCV) Bezier curves. For an effective result, the proposed model must satisfy the criteria for monotone curvature. Samples were provided to illustrate the monotone curvature distributions, and the aesthetically pleasing shapes of the curves were produced using their method. Furthermore, experiments showed that the curves that were created using this new method performed well in terms of both monotone curvature distribution and aesthetic shape.

An innovative generalised quartic H-Bezier basis function with four shape parameters was proposed by Hu et al. [7] to create the generalised quartic developable H-Bezier surfaces using control planes. Next, a comparison is made between the proposed model and the existing geometric modelling methods for developable surfaces.

3 Methodology

The full process involved in this study is shown in Figure 1 below:

Figure 1: The process of curve construction

A Selection of Sample Image

The process of reconstructing a curve started with a two-dimensional image involving x and y coordinates. This study has decided to reconstruct a two-dimensional image of a curve through the sample of an original image from [10] shown in Figure 2.

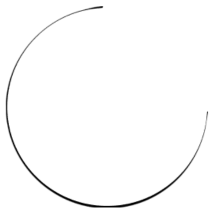

Figure 2: The image of a major curve from [10]

B Boundary Extraction

Several techniques were involved in the reconstruction process, which began with the boundary extraction of the image, the parameterisation process, and curve reconstruction. The scope of this study is to reconstruct the curve using uniform parameterisation. The first technique used in this study was extracting the boundary from the curve image. These boundary points from the curve image were selected for the next process, determining the control points to reconstruct the curve using a quartic Bezier curve. The boundary points were detected from the image of the curve by applying a boundary extraction algorithm using MATLAB software. Next, the boundary was extracted from the curve image as shown in Figure 3.

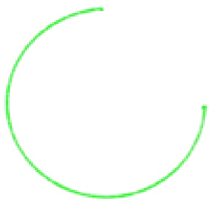

Figure 3: The boundary extraction of the curve

C Parameterisation Process

The initial process in this study was parameterisation. The reconstruction curve in this study utilised uniform parameterisation. The uniform parameterisation method was used to determine the parameter values, *t* for curve reconstruction.

Uniform parameterisation

Uniform parameterisation is a straightforward technique of parameterisation, and the parameters are evenly allocated in the range of $(0,1)$ [12]. Based on [11-13], the uniform parameterisation is given by:

$$t_i = \frac{i}{n},$$

where $0 \le i \le n-1$ and $t_0 = 0, t_n = 1$ (1)

4 Reconstruction of the curve

A Quartic Bezier Curve

A polynomial curve with many sets of control points is known as a Bezier curve. The parametric form of Bezier polynomial curves is as follows:

$$C(t) = \sum_{i=0}^{n} C_i B_i^n(t), t \in [0,1] \quad (2)$$

In this equation, the point C_i are the control points and the function $B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$ is the Bernstein Polynomial of degree n [6].

Then, the quartic Bezier curve by the t parameter with five control points is given by [2]:

$$C(t) = (1-t)^4 C_0 + 4(1-t)^3 t C_1 + 6t^2(1-t)^2 C_2 + 4(1-t)t^3 C_3 + t^4 C_4 \tag{3}$$

Where C_i, $0 \leq i \leq 4$ are the control points of the quartic Bezier.

A quartic Bezier curve is a higher degree of Bezier curve that is defined by the control points. The control points play an important role in shaping the curve. In this study, the control points have been used to generate a Bezier quartic equation as follows:

$$\begin{aligned} B_{0,4} &= (1-t)^4 \\ B_{1,4} &= 4t(1-t)^3 \\ B_{2,4} &= 6t^2(1-t)^2 \\ B_{3,4} &= 4t^3(1-t) \\ B_{4,4} &= t^4 \end{aligned} \tag{4}$$

The quartic Bezier curve with five control points is defined by C_0, C_1, C_2, C_3 and C_4. Given C_0 and C_1 are the endpoints of the curve, therefore $C_0 = P_0$ and $C_4 = P_4$ as shown in Figure 4.

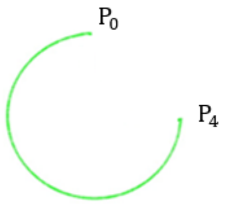

Figure 4: The endpoints of the curve

The control points of C_1, C_2 , C_3 are determined by using a technique from [14-15]. The quartic Bezier curve is written as $C(t) = \sum_{i=0}^{4} C_i B_i^4(t)$ and $t \, \epsilon \, [0,1]$ given by $B_i^4(t)$ is the ith quartic Berstein polynomial. To find the control points, we use the system of equations as follows:

$$\begin{aligned} \textstyle\sum_{i=0}^{4} C_i B_i^4(t_0) &= P_0 \\ \textstyle\sum_{i=0}^{4} C_i B_i^4(t_1) &= P_1 \\ \textstyle\sum_{i=0}^{4} C_i B_i^4(t_2) &= P_2 \\ \textstyle\sum_{i=0}^{4} C_i B_i^4(t_3) &= P_3 \\ \textstyle\sum_{i=0}^{4} C_i B_i^4(t_4) &= P_4 \end{aligned} \tag{5}$$

where P_i are the data points. Next, to solve the system equations for of C_1, C_2 , C_3, we let:

$$D = \begin{bmatrix} B_i^4(t_0) & \cdots & B_i^4(t_0) \\ \vdots & \ddots & \vdots \\ B_i^4(t_4) & \cdots & B_i^4(t_4) \end{bmatrix}, \qquad C = \begin{bmatrix} C_i \\ \vdots \\ C_i \end{bmatrix}, \qquad P = \begin{bmatrix} P_0 \\ \vdots \\ P_4 \end{bmatrix} \tag{6}$$

Then, $DC = P$, these equations can be written in a matrix form denoted as:

$$\begin{bmatrix} B_i^4(t_0) & \cdots & B_i^4(t_0) \\ \vdots & \ddots & \vdots \\ B_i^4(t_4) & \cdots & B_i^4(t_4) \end{bmatrix} \begin{bmatrix} C_i \\ \vdots \\ C_i \end{bmatrix} = \begin{bmatrix} P_0 \\ \vdots \\ P_4 \end{bmatrix} \tag{7}$$

To solve for C_1, C_2 , C_3 we defined the equation (8) to determine the new control points.

$$C = PD^{-1} \tag{8}$$

Next, the curve is constructed based on the new control points obtained from uniform parameterisation.

5 Program Codes

Step 1: Find the boundary extraction using MATLAB.
Step 2: Determine the endpoints of the sample curve from the boundary extraction.
Step 3: Find the parameter values t_0 , t_1, t_2 , $t_{3,} t_4$ using uniform parameterisation.
Step 4: Find the new control points.
Step 5: Reconstruct the curve.

Steps 3, 4, and 5 are using Wolfram Mathematica version 14.
Parametric Plot[{(1-t)^4 Q1x+4t (1-t)^3 t Q2x+6t^2 (1-t)^2 Q3x+4t^3 (1-t)Q4x+t^4 Q5x,(1-t)^4 Q1y+4t (1-t)^3 t Q2y+6t^2 (1-t)^2 Q3y+4t^3 (1-t)Q4y+t^4 Q5y},{t,0,1},PlotLegends->{"uniform"}]
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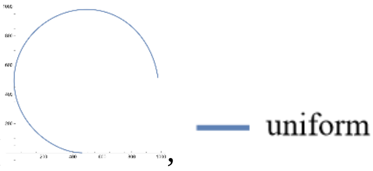
{ 

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6 Results and Discussion

A The Reconstruction of the Quartic Bezier Curve

The quartic Bezier curve with five (5) control points of a curve image was reconstructed, starting with the control points obtained. The result of the reconstruction of the image of the curve by using a quartic Bezier curve is shown in Figure 5 below.

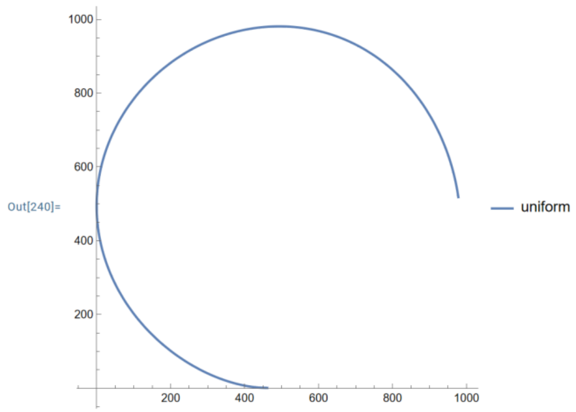

Figure 5: The reconstruction of the curve image using a quartic Bezier curve.

The finding from the reconstruction of the curve exhibits an improved curve with smooth curvature compared to the previous curve. The proposed technique from this study can be beneficial for the researchers and designers to reconstruct the curve in a significant process.

7 Conclusion

The properties, basis function for a Bezier curve, parameterisation process, and control points are the essential factors that contribute to the reconstruction of a quartic Bezier curve. Consequently, this study proposes the reconstruction of a higher Bezier curve with uniform parameterisation to improve the curve reconstruction process. Therefore, for future studies, researchers should enhance the parameterisation method or increase the number of segments for a better result.

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