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A Binary Logarithm Similarity Measure with Roughness Approximation of Rough Neutrosophic Set for COVID-19

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Abstract: Roughness measures for uncertainty data occur with less consideration since the data involve indeterminacy and inconsistency. The indeterminacy plus inconsistency can be solved by a rough neutrosophic set with roughness approximation. Therefore, a binary logarithm similarity measure for a rough neutrosophic set with roughness approximation was proposed in this research. A rough neutrosophic set was chosen as the uncertainty set theory information, which includes the upper and lower approximation with a boundary set approximation. The objectives of this research are to define a binary logarithm similarity measure for a rough neutrosophic set, to formulate the properties satisfied by the proposed similarity measure, and to develop a decision-making model by using a binary logarithm similarity measure for a case study (COVID-19). The roughness approximation was used in the derivation of the binary logarithm similarity measure. The proving result was finalized. Then, the derivation of binary logarithm similarity measures of a rough neutrosophic set was well defined. As a validation process, the similarity properties for identifying the most important priority group for COVID-19 vaccines were used such as age, health state, women, and job types. Following that, the decision-making for identifying the most important priority group for COVID-19 vaccines is presented.

Keywords: Binary logarithm, COVID-19, Rough neutrosophic set, Similarity measure

1 Introduction

Realistic practical problems in decision-making have many uncertainties and complexities [1]. Various uncertainty sets have developed distance and similarity measures, correlation coefficients, and aggregation operators applied for decision-making analysis. These operators are structured based on different uncertainty sets and aim to develop better solutions for decision-making problems. It is rational to choose the most dealing set and operator that can manage uncertain situations. The notion of fuzzy set theory (FS) was introduced by Zadeh [2]. This theory has been applied in many real-life applications to handle uncertainty. A FS theory allows the uncertainty of a set with a membership degree value between 0 and 1. This theory has achieved great success in various real applications to handle uncertainty but is unable to deal with certain problems due to inadequacy of parametrization tools [3].

Atanassov extended the FS to intuitionistic fuzzy sets (IFS) and stated that the IFS is a representation of both membership and non-membership uncertainty and the values are in the interval [0,1] [4]. However, IFS is unable to handle the indeterminate and inconsistent information that exists in a belief system. Then, Smarandache proposed a neutrosophic set (NS) which is the generalization theory of FS and IFS [5]. The NS is mainly concerned with indeterminate and inconsistent information. The NS consists of three membership functions which are truth-membership function, indeterminacy membership function, and falsity-membership function, with membership value [0,1], respectively. Then, Broumi defined rough neutrosophic (RNS) as a combination of rough set (RS) and NS [6]. This theory proposes to deal with problems involving uncertain, imprecise, incomplete, and inconsistent information existing in real-world problems. RNS is a powerful mathematical tool to deal with incompleteness since the structure of RS concerns with the boundary set approximation [7].

Similarity measure is important in the decision-making process, to solve the complex and uncertain nature of the problem resulting in strong and weak relationships between the attributes tested. The similarity measure for NS was studied extensively, such as distance–similarity measure [8], generalized similarity measure [9], Cosine similarity measure [10], hybrid binary logarithm similarity measure [11], and vector similarity measure [12]. There are also similarity measures structured by RNS. Cotangent and cosine similarity measure of RNS was proposed and applied in medical diagnosis [13]. Tri-complex RNS similarity measure is defined and solves the problem in multi-attribute decision-making [14]. Some RNS similarity measures such as variational coefficient similarity measure [1] and Pi-distance similarity measure applied in medical diagnosis [15]. All the similarity measures are defined by the mean relation between the lower and upper approximation sets. However, [16] proposed distance-oriented similarity measure of RNS by considering the roughness approximation instead of the mean value. Then, [17] studied the Cosine-roughness similarity measure for RNS.

The COVID-19 pandemic, also known as the Coronavirus Pandemic, has spread to various countries throughout the world. The entire world began to struggle with different generations of COVID-19 from the minute the announcement was made. Quick identification of COVID-19 patients is important to facilitate timely treatment and management of COVID-19 patients [18]. The diagnosis of this virus in the early stage is important for patients' quick recovery. The neutrosophic theory has contributed to many decision-making problems and also contributed to the early stage of COVID-19 [19]. A similarity measure of two neutrosophic soft sets based on Normalize Hamming distance and Normalized Euclidian distance was studied to estimate the possibility that an ill person having COVID-19 certain symptoms [18]. Others discussed patients' COVID-19 x-ray scans [20]. They combined neutrosophic logic in their model and used a real-world dataset to test their model. Others built a model that combined COVID-19's disruptive technologies for evaluating the pandemic virus [21].

Many researchers attempted to contribute more to the development of models for dealing with COVID-19 using different algorithms and mathematical tools. In this situation, RNS is also a useful tool to deal with uncertainty and complex problems. So, this research aims to propose a similarity measure in an RNS environment, named binary logarithm similarity measure. As a result, the formulation and properties of binary logarithm similarity measure based on roughness approximation of the RNS set are presented. Then, the verification process of the binary logarithm similarity measure was based on the roughness of RNS. Lastly, obtaining the best result while being prepared emphasizes the fact that the measure of roughness and binary logarithm similarity for RNS can be applied to case studies related to COVID-19.

2 Definitions of Terms and Concepts

The following are the definitions of terms and concepts used in this research. The roughness and binary logarithm similarity measure of the rough neutrosophic set will be developed based on the following assumption and notations.

A Assumption

All the set data must be in the equivalence relation.

B Notation

As depicted in Figure 1, the following notation was employed in this research.

Notation	Description
U	Universal set
$\underline{N}(A)$	Lower approximation of set A
$\overline{N}(A)$	Upper approximation of set A
$\underline{N}(B)$	Lower approximation of set B
$\overline{N}(B)$	Upper approximation of set B
$T_{\underline{N}(A)}$	Truth membership function for lower approximation of set A
$T_{\overline{N}(A)}$	Truth membership function for upper approximation of set A
$T_{\underline{N}(B)}$	Truth membership function for lower approximation of set B
$T_{\overline{N}(B)}$	Truth membership function for upper approximation of set B
$I_{\underline{N}(A)}$	Indeterminacy membership function for lower approximation of set A
$I_{\overline{N}(A)}$	Indeterminacy membership function for upper approximation of set A
$I_{\underline{N}(B)}$	Indeterminacy membership function for lower approximation of set B
$I_{\overline{N}(B)}$	Indeterminacy membership function for upper approximation of set B
$F_{\underline{N}(A)}$	Falsity membership function for lower approximation of set A
$F_{\overline{N}(A)}$	Falsity membership function for upper approximation of set A
$F_{\underline{N}(B)}$	Falsity membership function for lower approximation of set B
$F_{\overline{N}(B)}$	Falsity membership function for upper approximation of set B
$(\underline{N}(A), \overline{N}(A))$	Rough Neutrosophic Set A
$(\underline{N}(B), \overline{N}(B))$	Rough Neutrosophic Set B

Figure 1: List of the Notation

C Preliminaries

This section recalls the definitions used in this research.

Definition 1: Neutrosophic set [5]
Let X be a space of points (objects) with generic elements in X denoted by x; then the neutrosophic set $A(NS(A))$ is an object having form $A = [(x, T_A(x), I_A(x), F_A(x)) : x \in X]$, where the functions $T, I, F : X \to]^-0, 1^+[$ define, respectively, the truth-membership function, an indeterminacy membership function, and a falsity-membership function of the element $x \in X$ to set A with the condition:

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \tag{1}$$

Function $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$ since it is difficult to apply the neutrosophic set setting to practical problems.

Definition 2: Single-valued neutrosophic set [22]
Let X be a space of points (objects) with generic elements in X denoted by x. A single-valued neutrosophic set $A(SVNS(A))$ is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in $X, T_A(x), I_A(x), F_A(x) \in [0,1]$. A SVNS can be written as:

$$A = \{\langle x\ ;\ T_A(x), I_A(x), F_A(x)\rangle : x \in X\}. \tag{2}$$

Definition 3: Rough neutrosophic set [6]
Let U be a non-null set and R be an equivalence on U. Let A be a neutrosophic set in U with the membership function T_A, indeterminacy function I_A, and non-membership function F_A. The lower and upper approximations of A in the approximation *(U, R)* are denoted by $\underline{N}(A)$and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \{\langle x, T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x)\rangle : y \in |x|R, x \in y\}$$
$$\overline{N}(A) = \{\langle x, T_{\underline{N}(A)}(x), I_{\underline{N}(A)}(x), F_{\underline{N}(A)}(x)\rangle : y \in |x|R, x \in y\} \quad (3)$$

where,

$$T_{\underline{N}(A)}(x) = \wedge_{z\in[x]R} T_{(A)}(y), I_{\underline{N}(A)}(x) = \wedge_{z\in[x]R} I_{(A)}(y), F_{\underline{N}(A)}(y) = \wedge_{z\in[x]R} F_{(A)}(y),$$
$$T_{\overline{N}(A)}(x) = \vee_{z\in[x]R} T_{(A)}(y_j), I_{\overline{N}(A)}(x) = \vee_{z\in[x]R} I_{(A)}(y), F_{\overline{N}(A)}(x) = \vee_{z\in[x]R} F_{(A)}(y),$$
$$0 \le T_{\underline{N}(A)}(x) + I_{\underline{N}(A)}(x) + F_{\underline{N}(A)}(x) \le 3, \text{ and } 0 \le T_{\overline{N}(A)}(x) + I_{\overline{N}(A)}(x) + F_{\overline{N}(A)}(x) \le 3.$$

The $\wedge$ and $\vee$ denote "min" and "max" operators respectively, $T_{(A)}(y), I_{(A)}(y)$ and $F_{(A)}(y)$ are the membership, indeterminacy, and non-memberships of U with respect to A. From the above definition, it can be seen that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence classes of R if

$\underline{N}(A)= \overline{N}(A)$; i. e,
$$T_{\underline{N}(A)}(x) = T_{\overline{N}(A)}(x), I_{\underline{N}(A)}(x) = I_{\overline{N}(A)}(x), F_{\underline{N}(A)}(x) = F_{\overline{N}(A)}(x)$$

for any X belongs to U.

If *N(A)* is a rough neutrosophic set in *(U,R),* the rough complement of *N(A)* for these rough neutrosophic sets is denoted by $\sim N(A) = \left(\underline{N}(A)\right)^C, \left(\overline{N}(A)\right)^C$, where $\left(\underline{N}(A)\right)^C$ and $\left(\overline{N}(A)\right)^C$ are the complements of a neutrosophic set ($\underline{N}(A)$, $\overline{N}(A)$), respectively.

$$\sim N(A) = \left(\left(\underline{N}(A)\right)^C, \left(\overline{N}(A)\right)^C\right) = \{\langle \begin{matrix} x, ([F_{\underline{N}(A)}(x), 1 - I_{\underline{N}(A)}(x), T_{\underline{N}(A)}(x)], \\ [F_{\overline{N}(A)}(x), 1 - I_{\overline{N}(A)}(x), T_{\overline{N}(A)}(x)]) \end{matrix} \rangle : x \in U\} \quad (4)$$

Definition 4: Binary logarithm similarity measure for single-valued neutrosophic set [11]
Let $A = \langle x, (T_A(x), I_A(x), F_A(x))\rangle$ and $B = \langle x, (T_B(x), I_B(x), F_B(x))\rangle$ be any two single-valued neutrosophic sets (SVNS). The binary logarithm similarity measure between SVNSs A and B is defined as follows:

$$BL_1(A, B) = \frac{1}{n} \sum_{i=1}^{n} log_2 (2 - (\frac{1}{3} (|T_A(x) - T_B(x)| + |I_A(x) - I_B(x)| + |F_A(x) - F_B(x)|))) \quad (5)$$

Proposition 1: The binary logarithm similarity measure $BL_1(A, B)$ between any two SVNSs A and B satisfy the following properties [11]:

$(S1)$ $0 \le BL_1(A, B) \le 1$

$(S2)$ $BL_1(A, B) = 1$ if and only if $A = B$

$(S3)$ $BL_1(A, B) = BL_1(B, A)$

$(S4)$ If C is a SVNS in X and $A \subseteq B \subseteq C$ then $BL_1(A, C) \le BL_1(A, B)$ and $BL_1(A, C) \le BL_1(B, C)$.

Definition 5: Roughness measure for rough neutrosophic set [16]
Assume that A and B are any two rough neutrosophic sets in the universe of discourse X. ρ denotes the "roughness approximation" operator by rough approximation between the lower and upper approximation of rough neutrosophic set, A and B, while $|X|$ is the cardinality of the universal X, as follows:

$$\rho T_{N(A)}(x_j) = 1 - \left(\frac{T_{\underline{N}(A)}(x_j) + \left(T_{\overline{N}(A)}(x_j)\right)^C}{|X|}\right),\ \rho T_{N(B)}(x_j) = 1 - \left(\frac{T_{\underline{N}(B)}(x_j) + \left(T_{\overline{N}(B)}(x_j)\right)^C}{|X|}\right),$$

$$\rho I_{N(A)}(x_j) = 1 - \left(\frac{I_{\underline{N}(A)}(x_j) + \left(I_{\overline{N}(A)}(x_j)\right)^C}{|X|}\right),\ \rho I_{N(B)}(x_j) = 1 - \left(\frac{I_{\underline{N}(B)}(x_j) + \left(I_{\overline{N}(B)}(x_j)\right)^C}{|X|}\right),$$

$$\rho F_{N(A)}(x_j) = 1 - \left(\frac{F_{\underline{N}(A)}(x_j) + \left(F_{\overline{N}(A)}(x_j)\right)^C}{|X|}\right), \text{ and } \rho F_{N(B)}(x_j) = 1 - \left(\frac{F_{\underline{N}(B)}(x_j) + \left(F_{\overline{N}(B)}(x_j)\right)^C}{|X|}\right).$$

Such that

$$\rho T_{N(A)}(x_j), \rho I_{N(A)}(x_j), \rho F_{N(A)}(x_j), \rho T_{N(B)}(x_j), \rho I_{N(B)}(x_j), \rho F_{N(B)}(x_j) \in [0,1], \text{ and}$$

for $j = 1,2, \ldots, n$.

3 Methodology

The research methodology consists of three (3) phases of the development process as shown in Figure 2.

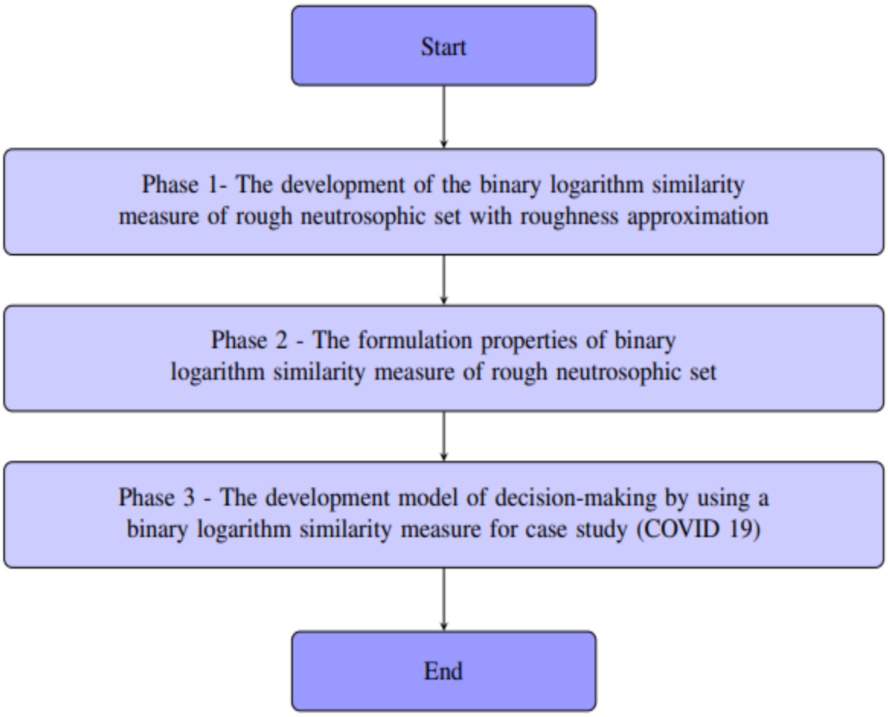

Figure 2: Methodology Phases

3.1 Phase 1: The development of the binary logarithm similarity measure of a rough neutrosophic set with roughness approximation.

A binary logarithm similarity measure (Definition 4) is generalized to a rough neutrosophic set with roughness approximation (Definition 5). A new definition is developed for the binary logarithm similarity measure of a rough neutrosophic set with roughness approximation.

3.2 Phase 2: The formulation properties of binary logarithm similarity measure based on roughness approximation of rough neutrosophic set.

The formulation is derived for a proposed binary logarithm similarity measure based on a roughness approximation of a rough neutrosophic set to prove Proposition 1.

3.3 Phase 3: The development of a decision-making model by using a binary logarithm similarity measure for a case study (COVID-19).

There are four (4) steps under Phase 3 for the verification process as shown in Figure 3.

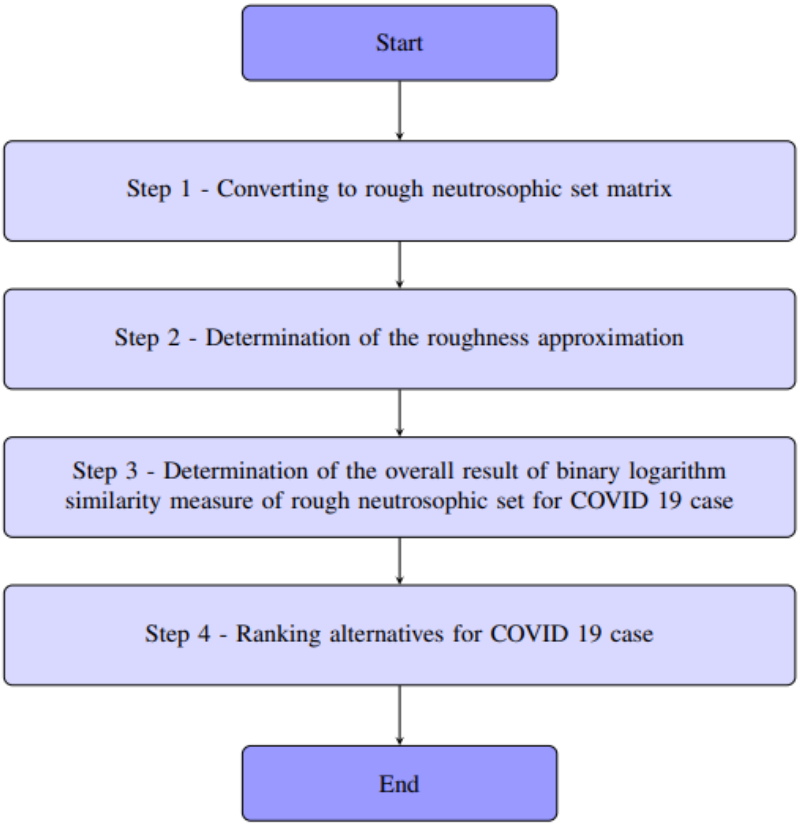

Figure 3: Verification Process

The explanation for each step is as follows:

Step 1: Converting to a rough neutrosophic decision matrix.
Decision makers consider the decision matrix with respect to i alternatives and j attributes in terms of rough neutrosophic numbers as shown in Figure 4.

$$D = \langle \underline{d}_{ij}, \overline{d}_{ij} \rangle_{mxn} =
\begin{array}{c|cccc}
 & A_1 & A_2 & A_n & \\
\hline
S_1 & \langle \underline{d}_{11}, \overline{d}_{11} \rangle & \langle \underline{d}_{12}, \overline{d}_{12} \rangle & \langle \underline{d}_{1n}, \overline{d}_{1n} \rangle & \\
S_2 & \langle \underline{d}_{21}, \overline{d}_{21} \rangle & \langle \underline{d}_{22}, \overline{d}_{22} \rangle & \langle \underline{d}_{2n}, \overline{d}_{2n} \rangle & \\
. & \cdots & \cdots & \cdots & \cdots \\
. & \cdots & \cdots & \cdots & \cdots \\
S_m & \langle \underline{d}_{m1}, \overline{d}_{m1} \rangle & \langle \underline{d}_{m2}, \overline{d}_{m2} \rangle & \langle \underline{d}_{mn}, \overline{d}_{mn} \rangle &
\end{array}$$

Figure 4: Rough Neutrosophic Decision Matrix [23]

Step 2: Determination of the roughness approximation
The development of the binary logarithm similarity measure theory is presented based on the roughness approximation.

Step 3: Determination of the overall result of binary logarithm similarity measure of a rough neutrosophic set for COVID-19 case.
The overall result of the binary logarithm similarity measure of a rough neutrosophic set is determined by using a proposed definition.

Step 4: Ranking alternatives for COVID-19 case.
Using a binary logarithm similarity measure of a rough neutrosophic set between each alternative and attribute, the ranking order of all alternatives determines the best alternative that can be selected with the highest similarity measure.

4 Results and Discussion

According to the research methodology, there are three (3) phases in achieving the objectives of this research.

4.1 Binary logarithm similarity measure of rough neutrosophic set.

Phase 1: The development of the binary logarithm similarity measure of a rough neutrosophic set with roughness approximation.

Definition 6: Binary logarithm similarity measure for a rough neutrosophic set.
Assume that A and B are any two rough neutrosophic sets in X as follows:

$A = \{\langle x_i; T_A(x_i), I_A(x_i), F_A(x_i)\rangle : x_i \in X\}$

$B = \{\langle x_i; T_B(x_i), I_B(x_i), F_B(x_i)\rangle : x_i \in X\}$

Then, a binary similarity measure between two rough neutrosophic sets A and B , $B_{RNS}(A, B)$ are defined as:

$$B_{RNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} log_2\,(2 - (\frac{1}{3}\,(|\rho T_A(x_i) - \rho T_B(x_i)| + |\rho I_A(x_i) - \rho I_B(x_i)| + |\rho F_A(x_i) - \rho F_B(x_i)|))) \quad (7)$$

where for any $(x_i = 1,2,3, \dots, n)$;

Here, $\rho T_A(x_i), \rho I_A(x_i), \rho F_A(x_i), \rho T_B(x_i), \rho I_B(x_i), \rho F_B(x_i) \in [0,1]$ is a roughness approximation of rough neutrosophic sets A and B (Definition 5).

Phase 2: The formulation properties of binary logarithm similarity measure based on roughness approximation of rough neutrosophic set.

The binary rough neutrosophic set $B_{RNS}(A, B)$ between two rough neutrosophic sets A and B satisfy the following properties:

Proposition 2:
$(S1)$ $\quad 0 \le B_{RNS}(A, B) \le 1$
$(S2)$ $\quad B_{RNS}(A, B) = 1$ if and only if $A = B$
$(S3)$ $\quad B_{RNS}(A, B) = B_{RNS}(B, A)$

$(S4)$ If C is a SVNS in X and $A \subseteq B \subseteq C$ then $B_{RNS}(A, C) \leq B_{RNS}(A, B)$ and $B_{RNS}(A, C) \leq B_{RNS}(B, C)$

Proof:

$(S1)$**:** From the definition of RNSs A and B,
$0 \leq \rho T_A(x_i) + \rho I_A(x_i) + \rho F_A(x_i) \leq 3$ and $0 \leq \rho T_B(x_i) + \rho I_B(x_i) + \rho F_B(x_i) \leq 3$
Then,
$0 \leq |\rho T_A(x_i) - \rho T_B(x_i)| + |\rho I_A(x_i) - \rho I_B(x_i)| + |\rho F_A(x_i) - \rho F_B(x_i)| \leq 3,$
$0 \leq \max\,(|\rho T_A(x_i) - \rho T_B(x_i)| + |\rho I_A(x_i) - \rho I_B(x_i)| + |\rho F_A(x_i) - \rho F_B(x_i)|) \leq 1$
Hence, $0 \leq B_{RNS}(A, B) \leq 1$

$(S2)$**:** For any two $RNS(A)$ and $RNS(B)$, if $A = B$, then the following relations hold for any.
$\rho T_A(x_i) = \rho T_B(x_i), \rho I_A(x_i) = \rho I_B(x_i)$ and $\rho F_A(x_i) = \rho F_B(x_i)$ according to which
$|\rho T_A(x_i) - \rho T_B(x_i)| = 0, |\rho I_A(x_i) - \rho I_B(x_i)| = 0, |\rho F_A(x_i) - \rho F_B(x_i)| = 0$
Hence, $B_{RNS}(A, B) = 1$
Conversely,
If $B_{RNS}(A, B) = 1$, from the result we have,
$|\rho T_A(x_i) - \rho T_B(x_i)| = 0, |\rho I_A(x_i) - \rho I_B(x_i)| = 0, |\rho F_A(x_i) - \rho F_B(x_i)| = 0$, then,
$\rho T_A(x_i) = \rho T_B(x_i), \rho I_A(x_i) = \rho I_B(x_i), \rho F_A(x_i) = \rho F_B(x_i)$.
Hence $A = B$.

$(S3)$**:** It is obvious that,
$|\rho T_A(x_i) - \rho T_B(x_i)| = |\rho T_B(x_i) - \rho T_A(x_i)|, |\rho I_A(x_i) - \rho I_B(x_i)| = |\rho I_B(x_i) - \rho I_A(x_i)|$, and
$|\rho F_A(x_i) - \rho F_B(x_i)| = |\rho F_B(x_i) - \rho F_A(x_i)|$.
Therefore, $B_{RNS}(A, B) = B_{RNS}(B, A)$
$(S4)$**:** For $A \subseteq B \subseteq C$.
$\rho T_A(x_i) \leq \rho T_B(x_i) \leq \rho T_C(x_i), \rho I_A(x_i) \leq \rho I_B(x_i) \leq \rho I_C(x_i), \rho F_A(x_i) \leq \rho F_B(x_i) \leq \rho F_C(x_i)$
For $x \in X$.

Since,
$|\rho T_A(x_i) - \rho T_B(x_i)| = |\rho T_A(x_i) - \rho T_C(x_i)|,$
$|\rho T_B(x_i) - \rho T_C(x_i)| = |\rho T_A(x_i) - \rho T_C(x_i)|,$
$|\rho I_A(x_i) - \rho I_B(x_i)| = |\rho I_A(x_i) - \rho I_C(x_i)|,$
$|\rho I_B(x_i) - \rho I_C(x_i)| = |\rho I_A(x_i) - \rho I_C(x_i)|,$
$|\rho F_A(x_i) - \rho F_B(x_i)| = |\rho F_A(x_i) - \rho F_C(x_i)|,$
$|\rho F_B(x_i) - \rho F_C(x_i)| = |\rho F_A(x_i) - \rho F_C(x_i)|,$

Therefore, $B_{RNS}(A, C) \leq B_{RNS}(A, B)$ and $B_{RNS}(A, C) \leq B_{RNS}(B, C)$
The proof is complete.

Phase 3: Empirical Example: COVID-19

The research data for the empirical example is from [24]. This research focuses on identifying the most important priority group for the COVID-19 vaccine. In this case study, there are three (3) experts which are expert 1 $(E1)$, expert 2 $(E2)$, and expert 3 $(E3)$. Meanwhile, there are six (6) alternatives which are elderly people $(A1)$, people with health problems $(A2)$, pregnant and breastfeeding women $(A3)$, health workers and people who have close contact with patients $(A4)$, healthy and young people $(A5)$, and children and young adults $(A6)$ to assume that each vaccine is more suitable than others for specific groups. There are also priority which consists of four (4) data which are age $(C1)$, health state $(C2)$, women $(C3)$, and job kinds $(C4)$.

Considering the third phase of the methodology, a similarity result for empirical example was obtained as follows:

Step 1: Converting to a rough neutrosophic decision matrix.
Following that, set A is a relation between expert and priority while set B is a relation between alternative and priority. The converting data is shown in Table 1 and Table 2, respectively.

Table 1: Relation between the expert and priority groups (set A)

A	$C1$	$C2$	$C3$	$C4$
$E1$	$\langle(0.2,0.5,0.5),(0.3,0.4,0.4)\rangle$	$\langle(0.2,0.5,0.5),(0.3,0.4,0.4)\rangle$	$\langle(0.2,0.5,0.5),(0.4,0.2,0.3)\rangle$	$\langle(0.3,0.4,0.4),(0.4,0.3,0.3)\rangle$
$E2$	$\langle(0.2,0.4,0.4),(0.3,0.4,0.4)\rangle$	$\langle(0.4,0.3,0.3),(0.2,0.3,0.5)\rangle$	$\langle(0.1,0.5,0.5),(0.2,0.5,0.4)\rangle$	$\langle(0.5,0.2,0.2),(0.2,0.2,0.5)\rangle$
$E3$	$\langle(0.2,0.4,0.4),(0.3,0.4,0.4)\rangle$	$\langle(0.4,0.3,0.3),(0.3,0.4,0.4)\rangle$	$\langle(0.3,0.4,0.4),(0.2,0.5,0.5)\rangle$	$\langle(0.2,0.5,0.4),(0.1,0.5,0.5)\rangle$

Table 2: Relation between the alternative and priority groups (set B)

B	$C1$	$C2$	$C3$	$C4$
$A1$	$\langle(0.3,0.5,0.5),(0.3,0.5,0.5)\rangle$	$\langle(0.2,0.6,0.6),(0.3,0.7,0.6)\rangle$	$\langle(0.1,0.6,0.6),(0.3,0.7,0.6)\rangle$	$\langle(0.2,0.6,0.6),(0.3,0.7,0.6)\rangle$
$A2$	$\langle(0.4,0.4,0.4),(0.4,0.5,0.5)\rangle$	$\langle(0.3,0.4,0.4),(0.4,0.5,0.5)\rangle$	$\langle(0.3,0.6,0.5),(0.4,0.6,0.6)\rangle$	$\langle(0.3,0.6,0.5),(0.4,0.6,0.6)\rangle$
$A3$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.8)\rangle$	$\langle(0.5,0.8,0.7),(0.1,0.8,0.8)\rangle$	$\langle(0.5,0.3,0.3),(0.6,0.3,0.3)\rangle$	$\langle(0.2,0.6,0.6),(0.3,0.7,0.6)\rangle$
$A4$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.1)\rangle$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.8)\rangle$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.8)\rangle$	$\langle(0.4,0.4,0.4),(0.5,0.4,0.4)\rangle$
$A5$	$\langle(0.2,0.5,0.5),(0.3,0.6,0.6)\rangle$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.8)\rangle$	$\langle(0.1,0.7,0.6),(0.2,0.7,0.7)\rangle$	$\langle(0.2,0.6,0.5),(0.3,0.6,0.5)\rangle$
$A6$	$\langle(0.2,0.5,0.5),(0.3,0.6,0.6)\rangle$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.8)\rangle$	$\langle(0.1,0.8,0.7),(0.1,0.,8\ 0.8)\rangle$	$\langle(0.1,0.8,0.7),(0.1,0.8,0.8)\rangle$

Step 2: Determination of roughness approximation for rough neutrosophic set.
By using Definition 5 and as equation (4), the roughness approximation is determined for each of the relations involved. For example, the roughness measure, for relation A between $E1$ and $C1$ was calculated as follows:

i) $\rho T_{N(A)}(x_j) = 1 - \left(\frac{T_{\underline{N}(A)}(x_j)+\left(T_{\overline{N}(A)}(x_j)\right)^C}{|X|}\right) = 1 - \left(\frac{0.2+0.4}{|4|}\right) = 0.9$

ii) $\rho I_{N(A)}(x_j) = 1 - \left(\frac{I_{\underline{N}(A)}(x_j)+\left(I_{\overline{N}(A)}(x_j)\right)^C}{|X|}\right) = 1 - \left(\frac{0.5+0.6}{|4|}\right) = 0.7$

iii) $\rho F_{N(A)}(x_j) = 1 - \left(\frac{F_{\underline{N}(A)}(x_j)+\left(F_{\overline{N}(A)}(x_j)\right)^C}{|X|}\right) = 1 - \left(\frac{0.5+0.3}{|4|}\right) = 0.8$

Then, by using the same equation and definition, the roughness measure for all membership function for each relation A and relation B for was calculated. The roughness measure for set A is shown as $RNS(A)$ in Table 3 meanwhile the roughness measure for set B is shown as $RNS(B)$ in Table 4.

Table 3: Roughness approximation set A

A	$C1$	$C2$	$C3$	$C4$
$E1$	⟨0.9,0.7,0.8⟩	⟨0.9,0.7,0.8⟩	⟨0.8,0.8,0.7⟩	⟨0.8,0.5,0.7⟩
$E2$	⟨0.8,0.7,0.8⟩	⟨0.7,0.7,0.8⟩	⟨0.7,0.7,0.8⟩	⟨0.8,0.7,0.8⟩
$E3$	⟨0.8,0.78,0.8⟩	⟨0.7,0.7,0.8⟩	⟨0.7,0.7,0.8⟩	⟨0.8,0.7,0.8⟩

Table 4: Roughness approximation set B

B	$C1$	$C2$	$C3$	$C4$
$A1$	⟨0.7,0.7,0.7⟩	⟨0.7,0.7,0.7⟩	⟨0.7,0.7,0.7⟩	⟨0.7,0.7,0.8⟩

$A2$	⟨0.7,0.7,0.7⟩	⟨0.7,0.7,0.7⟩	⟨0.7,0.7,0.7⟩	⟨0.8,0.7,0.8⟩
$A3$	⟨0.7,0.7,0.8⟩	⟨0.8,0.7,0.8⟩	⟨0.8,0.7,0.7⟩	⟨0.8,0.7,0.8⟩
$A4$	⟨0.8,0.7,0.8⟩	⟨0.8,0.7,0.8⟩	⟨0.8,0.8,0.8⟩	⟨0.8,0.8,0.8⟩
$A5$	⟨0.8,0.8,0.8⟩	⟨0.8,0.8,0.8⟩	⟨0.8,0.7,0.7⟩	⟨0.7,0.7,0.8⟩
$A6$	⟨0.8,0.7,0.8⟩	⟨0.8,0.7,0.8⟩	⟨0.8,0.7,0.8⟩	⟨0.8,0.7,0.7⟩

Step 3: Determination of the overall result of binary logarithm similarity measure of rough neutrosophic set.
A proposed binary logarithm similarity measure (Definition 6) was used to determine the similarity measure for the relation between alternatives and attributes. For example, the similarity measure for $E1$ and $A1$ was calculated as follows:

$$B_{RNS}(E1, A1)$$
$$= \frac{1}{4}\sum_{i=1}^{4} log_2\left(2 - \left(\frac{1}{3}\left(|\rho T_A(x_i) - \rho T_B(x_i)| + |\rho I_A(x_i) - \rho I_B(x_i)| + |\rho F_A(x_i) - \rho F_B(x_i)|\right)\right)\right)$$
$$= \frac{1}{4}\Bigg[log_2\left(2 - \left(\frac{1}{3}\left(|0.9 - 0.7| + |0.7 - 0.7| + |0.8 - 0.7|\right)\right)\right) + log_2\left(2 - \left(\frac{1}{3}\left(|0.9 - 0.7| + |0.7 - 0.7| + |0.8 - 0.7|\right)\right)\right) + log_2\left(2 - \left(\frac{1}{3}\left(|0.8 - 0.7| + |0.8 - 0.7| + |0.7 - 0.7|\right)\right)\right) + log_2\left(2 - \left(\frac{1}{3}\left(|0.8 - 0.7| + |0.5 - 0.7| + |0.7 - 0.8|\right)\right)\right)\Bigg]$$
$$= 0.9503$$

Then, by using the same definition, the similarity measure for relation A and relation B was calculated and shown in Table 5.

Table 5: Similarity measure for relation between alternatives and attributes

$B_{RNS}(A, B)$	$A1$	$A2$	$A3$	$A4$	$A5$	$A6$
$E1$	0.9503	0.9535	0.9535	0.9537	0.9503	0.9488
$E2$	0.9710	0.9700	0.9669	0.9695	0.9710	0.9655
$E3$	0.9792	0.9814	0.9780	0.9822	0.9792	0.9746

Step 4: Ranking the alternatives.
Based on the result of the similarity measure shown in Table 5, the similarity value nearest to one indicates the best-ranking alternative. The summary result indicates the similarity average between experts for each alternative. The final ranking is summarized in Table 6.

Table 6: The ranking for each alternative

Alternative	Similarity measure $B_{RNS}(A, B)$	Ranking
$A1$	0.9668	3
$A2$	0.9683	2
$A3$	0.9661	4
$A4$	0.9685	1
$A5$	0.9638	5
$A6$	0.9623	6

Based on the ranking for each alternative, health workers with people with close contact with patients ($A4$) is the most important priority group for COVID-19 vaccines. Followed by people with health problems ($A2$), elderly people ($A1$), pregnant and breastfeeding women ($A3$), health workers, healthy and young people ($A5$), and children and young adults ($A6$).

Comparative results for the proposed binary logarithm similarity measure for the rough neutrosophic set and [24] is shown in Table 7.

Table 7: The comparative results

Hybrid Similarity Measure	Ranking Order
Proposed $B_{RNS}(A, B)$	$A4 < A2 < A1 < A3 < A5 < A6$
Neutrosophic AHP and TOPSIS, [24]	$A2 < A4 < A1 < A3 < A5 < A6$

According to the comparative results, the proposed binary logarithm similarity measure outcome revealed a slightly different outcome from other existing outcomes of the selection results of the most important priority group for COVID-19 vaccines. Forthwith, the result for the proposed binary logarithm similarity measure provided a new alternative result for identifying the most important priority group for COVID-19 vaccines since it is being influenced by the roughness approximation determination for the lower and upper approximation. Circumstantially, the result for the similarity measure is a less roughness measure of information. In summary, the roughness approximation has a significant impact on the relationship of information, especially for similarity measures. The binary logarithm similarity measure for a rough neutrosophic set is to find the closest result to one which indicates the attributes according to their alternative.

5 Conclusion

In this research, roughness and binary logarithm similarity measure for RNS was introduced. The lower and upper approximation of the RNS gave the accuracy and roughness value between the information given. Meanwhile, similarity was used for the incomplete of information. The proposed method was compared with the existing method under a neutrosophic environment in identifying the most priority group for COVID-19 vaccines, and results show that the proposed method is more acceptable because, in the first phase, the measurement of roughness was done. So, the data may have less roughness for lower and upper approximation.

The research objectives were all achieved. The research has successfully proposed a definition for binary logarithm similarity measure for RNS and the results for determination has been verified by close to one as predicted. The second objective has been proven in Phase 2 in Proposition 2. The third objective was reached in the implementation where the formula of the roughness can be used to find a similarity result between alternatives and criteria. The result obtained from the work indicated the importance of classification and ranking, as priority was given to people with close contact with patients $(A4)$. The less roughness measure was used in this research. Roughness is suitable to deal with lower and upper approximation conditions. This tool will help society and organizations to solve selection problems efficiently since each alternative or project will have different costs, benefits, and risks. Hence, for future studies, it can be done by including more groups as well as more experts.

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Conflict of Interest Statement

The authors agree that this research was conducted in the absence of any self-benefits, commercial or financial conflicts and declare the absence of conflicting interests with the funders.

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