The Effect of Sidewall Friction on Complex Channel Geometry using LABSWE™

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ABSTRACT

Lattice Boltzmann Model for Shallow Water Equation with Turbulence Modeling (LABSWE™) is used to study the flow patterns of sidewall friction effects. The lattice Boltzmann method (LBM) approach in recovery the macroscopic governing equation which is shallow water equation from the microscopic flow behavior of particle movement as described by kinetic theory is explored. With the solution of force term to be used in lattice Boltzmann equation, the boundary condition of LBM is explored. With the use of bed and wall friction coefficients, the importance of Manning's coefficient in determining the outcome of flow patterns simulation is explained. For model verification, the model represents a straight channel with a circular cavity attached to it. The result of this simulation includes the water circulation patterns, cross-section of average velocity distribution, and water depth. For validation, the cross-sections of the model in term of velocity vectors are compared against alternative numerical and experimental data.

Keywords: *Lattice Boltzmann Methods, shallow water equations, semi-slip boundary, Manning's coefficient, sidewall friction, flow patterns.*

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Introduction

In studying flow patterns of a complex geometry channel, sidewall friction is an important catalyst besides bed topography, either in the real situation or simulations. Due to the problem of non-uniform velocity distribution effect from free surface and secondary flow in three-dimensional open channel, the sidewall friction is always neglected. It has been proved that the shear stress distribution on the wall in a planar and axially symmetric flow can be calculated from force balance if it is uniform along the wet perimeter and shear stress [1]. The main key factor in Lattice Boltzmann Method (LBM) is its implementation in parallel computations which is quite easy and comparable. A test case proposed to eliminate the statistical noise commonly found in Lattice Gas Automata (LGA), thus making simulation that demands much less computer time [2]. This makes LBM a promising and reliable method in recent era of modern computational hardware. Thus, LBM is a promising method to analyze the shallow water situations that usually consists of source terms made of external forces such as wind shear stress, free surface and most importantly from the interaction between the fluids and bed topography.

In this paper, the effect of sidewall friction on complex channel geometry that represents a lake attached to a river is simulated by Lattice Boltzmann Model for Shallow Water Equation with Turbulence Modeling (LABSWE™). Bed and wall friction are varied and different Manning's coefficients are used to have different effects on the flow patterns. The result of simulations are analyzed and discussed. In the simulations, the physical properties of water to be compared are water flow, height, and velocities distribution with regards to different friction coefficients.

LABSWE™

a. Governing Equations

LBM consists of two major steps, streaming and collision steps [3]. These two steps are combined into lattice Boltzmann equation proving that LBM is a simple yet efficient method. The Lattice Boltzmann equation is given by [3]:

$$f_\alpha(x + e_\alpha \Delta t, t + \Delta t) - f_\alpha(x,t) = -\frac{1}{\tau}\left(f_\alpha - f_\alpha^{eq}\right) + \frac{\Delta t}{6e^2} e_{\alpha i} F_i, \quad \text{(Eq. 1)}$$

Where, f_α = the distribution function of particle; f'_α = the value of f_α before the streaming; $e = \Delta x / \Delta t$, Δx is the lattice size; F_i = component of the force in i direction; and e_α = velocity vector of a particle in a link. Eq. 1 is the mostly used form of lattice Boltzmann model in modeling fluid flows.

b. Force term in LABSWE™

In LABSWE™, in order to make it mimic the real sidewall friction in a simulation, the boundary condition must be the semi-slip boundary. To accomplish the semi-slip boundary condition, the shear stress of the wall is incorporated into the force term,. It represents a natural and simple way to solve the lattice Boltzmann equation for semi-slip boundary condition at the boundary nodes. The F_i is given by [3]:

$$F_i = -gh\frac{\partial z_b}{\partial x_i} + \frac{\tau_{wi}}{\rho} + \frac{\tau_{bi}}{\rho} + \frac{\tau_{fi}}{\rho} \quad \text{(Eq. 2)}$$

Where, τ_{wi} = wind shear stress; τ_{bi} = bed shear stress; and τ_{fi} = wall shear stress.

c. Manning’s coefficient

In simulating sidewall friction of flow, Manning’s coefficient becomes the key factor. There are two variables affecting the flow patterns with regard to Manning’s coefficients: 1) coefficients of bed frictions, C_b; 2) coefficients of wall frictions, C_f.The value of Manning’s coefficients are as follow:

$$C_b = \frac{g n_b^2}{h^{1/3}}, \quad \text{(Eq. 3)}$$

$$C_f = g\frac{n_f^2}{h^{1/3}}, \quad \text{(Eq. 4)}$$

Where, n_b = Manning’s coefficient for bed; n_f = Manning’s coefficient for wall. The Manning’s coefficients used in this research are showed below.

Table 1: Coefficient for bed frictions

Simulation	n_b	Material
1	0.035	Earth channel – stony
2	0.01	Glass
3	0.05	Floodplains - light brush
4	0.011	Brass

Table 2: Coefficient for wall frictions

Simulation	n_f	Material
1	0.035	Earth channel – stony
2	0.01	Glass
3	0.015	Brickwork
4	0.075	Floodplains - heavy brush

d. Model Setup

A test case to analyze the flow patterns in a channel with sidewall friction is simulated by LABSWE™ [4]. The model simulates the effect of turbulent flow with sidewall friction in a river attached to a lake (a case involving complicated geometry). The results obtained by LABSWE™ are compared against experimental results [4]. The bed friction coefficient of 0.045 represent materials of light floodplain-light brush is used in this model. The rectangular channel is 7m wide and 19m long. A 3.15m radius circular sidewall cavity is located on the right side of the channel such in Figure 1.

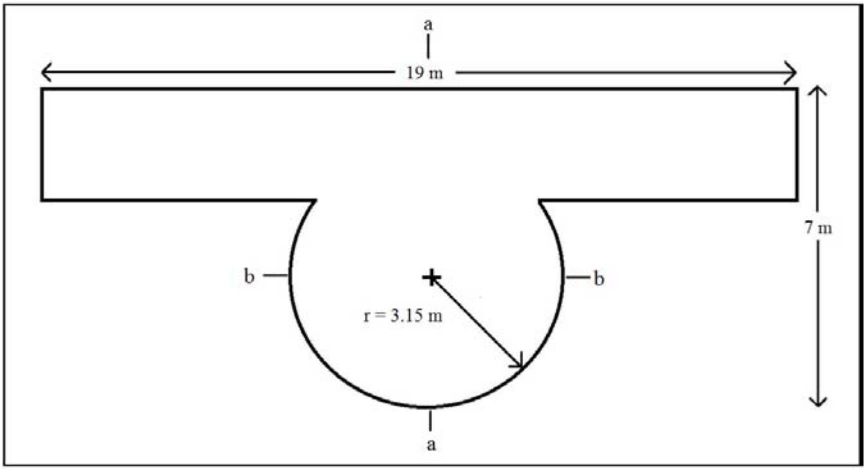

Figure 1: The shape of open-channel with circular sidewall cavity in top view

Throughout the computation, flow velocity components of u = 0.25m/s and v = 0m/s and water depth, h = 0.25m are imposed at the inflow and outflow boundaries.

A 190 × 70 lattice with grid space of Δx is used. A semi-slip boundary condition with surface roughness coefficient C_f=0.0045 is utilized at the solid walls. The relaxation time τ = 0.6 and the Smagorinsky constant C_s = 0.3 are applied [5]. A time step Δt=0.025s satisfies the stability criteria and hence is used in the model. The u components at a-a cross sections and v components at *b-b* cross sections will be compared with compared against simulation and experimental results obtained by Kuipers and Vreugdenhil.

Result and Discussion

The initial stage of simulation is carried out with semi-slip boundary conditions. After that several tests were done to ensure the program conditions are stable or otherwise. A problem arises where the vectors plotted are slightly diverged due to semi-slip boundary conditions implementation. Thus, this problem handled by using the slip boundary conditions rather than the semi-slip boundary conditions.

10000th iteration from different Manning's coefficient value in n_f and n_b are simulated. The result obtained are shown in the figures (2-5).

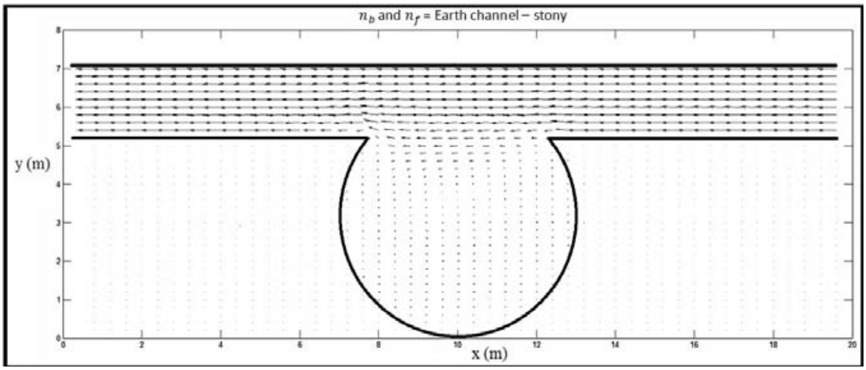

Figure 2: Velocity vector of Simulation 1

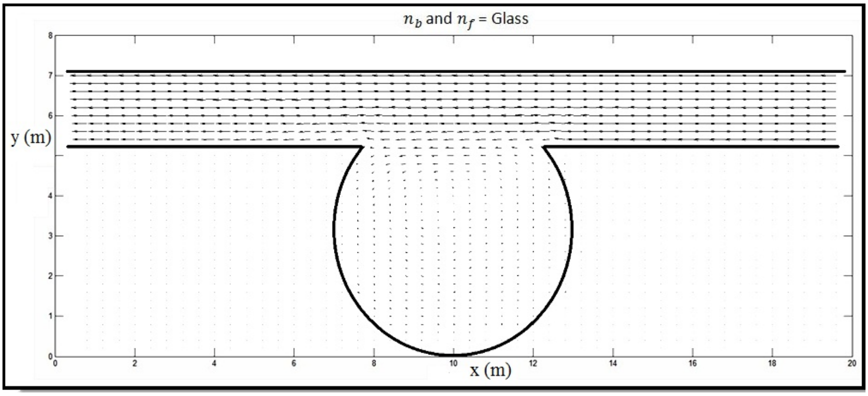

Figure 3: Velocity vector of Simulation 2

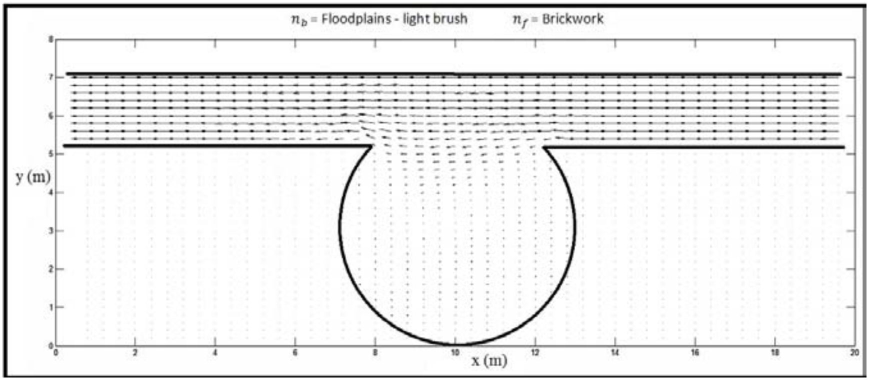

Figure 4: Velocity vector of Simulation 3

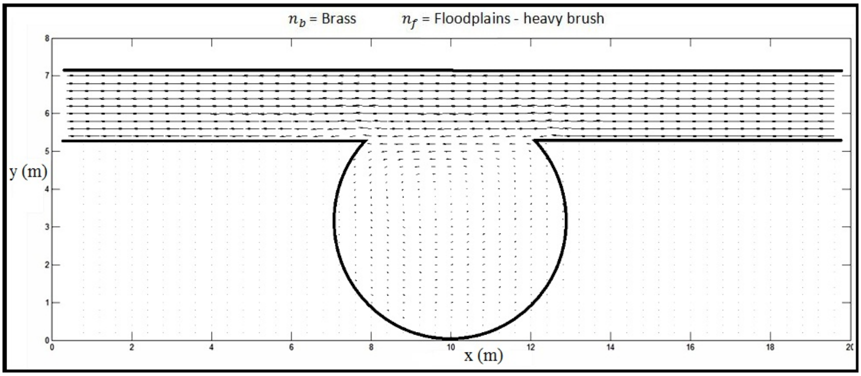

Figure 5: Velocity vector of Simulation 4

The vectors in the figures show a stable flow was achieved. It may not be an obvious comparison in the velocity vectors figures as the vectors inside the circular cavity likely to vanish. Thus, it is suggested that streamline figures may be a better view for flow comparisons. For validation procedure, the u and v components along *a-a* and *b-b* cross sections in the simulations results using LBM are compared against Kuipers and Vreugdenhil experiment data in Figures 6 and 7.

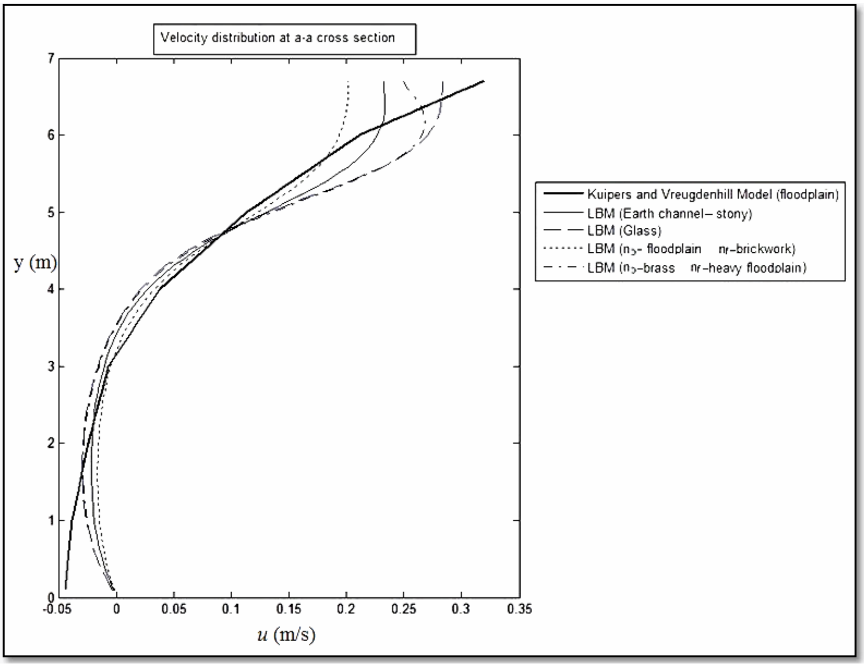

Figure 6: Comparison of *u* at *a-a* cross sections

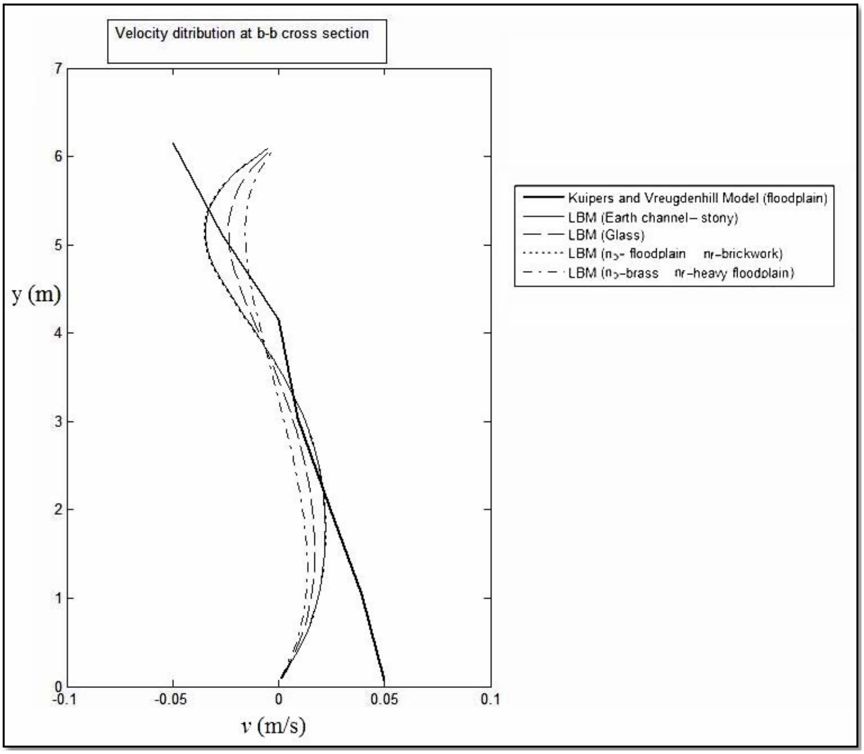

Figure 7: Comparison of *v* at *b-b* cross sections

Analysis for the relative error for velocity values of u components along the a-a cross sections was calculated by comparing the Simulation 3 of LABSWE™ with experimental data. Simulation 3 is chosen because it has the same materials of Manning's properties with the experimental data. The result obtained showed in Table 3.

Table 3: Relative error of *u* between LABSWE™ and experimental data

	Value
LABSWE™	0.0891
Experimental Data	0.104
Error	0.0129

The error is compared at point of y= 6.7m, 6m, 5m, 4m, 3m, 2m, and 1m. For experimental data the *u* mean values is 0.104 and for LABSWE™ the *u* mean values is 0.0891. Thus the error is 0.0129 or calculated at 12.4 percent. Both of the method are not showing a large difference in term of accuracy.

Conclusion

In conclusion, this test proves that a semi-slip boundary conditions with an addition to force term F_i can solve the shallow water problem regarding a complex channel geometry. The analysis finally gives us a better perspective that LABSWE™ performs better than Kuipers and Vreugdenhil model in predicting the flow patterns of an attached sidewall cavity with open channel.

The above figures (6 and 7) showed us two different analyses that should be taken into account for finding the current model's accuracy. First, there are comparisons between LBM simulations with different Manning's coefficient value. Both of u and v components in cross sections *a-a* and *b-b* gave good results. Secondly, the performance analysis between finite-difference in method [4] and LBM was also generated. There are slight differences in the u components where the curve is nearly the same and that because the alternative method used a different material of wall and bed friction coefficient. The v component comparison gives a less accurate prediction near the wall of circular cavity that caused by the unstable boundary conditions.

Recommendation

In simulating the test case, it is recommended to use a high performance computer with enormous processor capacity to save time. For the water flow pattern, the figures (2-5) do not give an obvious comparison to the naked eyes. Thus, streamline figures for water flow pattern need to be considered as it reviews the pattern in an obvious and clear way. Lastly, further research should be carried out to compare all the simulation results with the same data using alternative methods.

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