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# Furnace Modelling Using State Space Representation

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**ABSTRACT**

*The state space modeling approach was developed to cope with the demand and performance due to the increase in system complexity, which may have multiple inputs and multiple outputs (MIMO). This approach is based on time-domain analysis and synthesis using state variables. This paper describes the development of a state space representation of a furnace system of a combined cycle power plant. Power plants will need to operate optimally so as to stay competitive, as even a small improvement in energy efficiency would involve substantial cost savings. Both the quantitative and qualitative analyses of the state space representation of the furnace system are discussed. These include the responses of systems excited by certain inputs and the structural properties of the system. The analysis on the furnace system showed that the system is bounded input and bounded output stable, controllable and observable. In practice, the state space formulation is very important for numerical computation and controller design, and can be extended for time-varying systems.*

***Keywords**: State space model, Power generation plant, state variables, multivariable system*

## Introduction

The design of mathematical models of complex real-world systems is essential in many fields of science and engineering. The need for new approach and philosophies in modeling and control of complex industrial

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systems are much influenced by the recent advances in information technology, increased market competition, the demand for low cost operation and energy efficiency. In the electricity industry for example, power generation plants will need to operate optimally in order to stay competitive, as even a small improvement in energy efficiency would involve substantial cost savings. For a large complex system such as the power generation plants, it is useful to decompose the system into subsystems or components that can be analyzed and understood separately. The physical structure of the system often suggests suitable subdivision. To analyze such system, it is essential to reduce the complexity of the mathematical expressions and to resort to computers for most of the tedious computations involved in the analysis. Thus, the state space approach is best suited from this viewpoint. This approach is based on time-domain analysis and synthesis using state variables. In fact, the most important advantage of the crisp state space model is that the system dynamic properties are condensed in the model [1].

In general, a power generation plant consists of a gas turbine, a boiler and a steam turbine [2-3]. This paper focuses on the furnace system of a combined cycle power plant, which is regarded as constituents in a heat treatment system of a boiler. In order to increase the furnace thermal efficiency, it is important and necessary to understand the physical phenomena of the combustion process in a furnace system. Investigating the dynamics of power plant requires models with precise representation of plant components [2]. These models are used to build plant simulators, from which control strategies may be assessed. Many different types of models have been proposed to predict temperature and flux distribution in large furnaces. With respect to structural complexity, they may be classified as zero dimensional models, one-dimensional models and three-dimensional models [2]. Of these models, one-dimensional models are most suited for boiler simulation codes, since they may incorporate fundamental phenomena of combustion processes. Besides, the gas properties are assumed uniform on a horizontal plane. This agrees well with real boiler furnaces' behaviour where mixing of incoming fuel takes place in horizontal planes. Some aspects in the development of furnace model and control had been discussed [4-6]. The qualitative modeling of a dynamical process using system identification based on a gas furnace with single input and single output is also illustrated in Sugeno and Yasukawa [4]. The analysis of such thermal system is often very complicated. The characterizing equations are generally a set of partial differential equations, with nonlinearity arising due to convection of momentum in the flow, variable properties and radiative transport. However, approximation and idealizations are used to simplify these equations, resulting in algebraic and ordinary differential equations for many practical situations. Thus, it is assumed that the system can be represented by a lumped-parameter model. Lumped models are very useful in early stages of system analysis and also in verifying more complex computer models, as long as such models retain the basic dynamic characteristics of the original systems.

The rest of the paper is organized as follows. Section 2 describes the process involved in a furnace system of a combined cycle power plant, followed by the main assumptions used to construct the mathematical model in section 3. The mathematical model of the furnace system is presented in section 4. Subsequently, section 5 describes the development of the state space model of the furnace system. In order to understand the dynamic behaviour of the state space model of the furnace system, both the quantitative and qualitative analyses are discussed in section 6. Finally, section 7 draws some conclusion from the presented work.

## The Furnace System

A furnace is an item of process or thermal equipment isolated from the surroundings by an enclosure. It is used to obtain heat from other forms of energy or to transfer heat to a material to be heat-treated as required by a particular technological process [7]. It is one of the important subsystems in a boiler of a combined cycle power plant. Other subsystems are risers, drum, superheater with attemporator and reheater [3]. The schematic diagram of the boiler system is shown in Figure 1. A boiler consists of two main parts [8]; the first part is the furnace which converts the chemical energy of the fuel into heat and the second part is the boiler shell or tubes in which the water is converted into steam by means of the heat generated in the furnace.

Based on a typical drum-type boiler shown in Figure 1, the feedwater is supplied to the drum where part of the water is evaporated. The water flows into the downcomers and enters the risers. In the risers, the heat from the furnace or combustion chamber is used to increase the water temperature and eventually to cause evaporation. Thus the circulation of water, steam, and water and steam mixture takes place in the drum, the downcomers and the risers. Steam generated in the risers is separated in the drum from where it flows through the superheater on to the high-pressure turbines. It may be recycled to the boiler in the reheater where its energy content is increased. Desuperheating spray water is introduced in the superheater for control of main steam temperature. As for the combustion process path, the risers absorb radiant heat in the furnace. The hot gases leaving the furnace transfer heat by radiation and convection to the superheater. The heat is then transferred by convection to the reheater and finally to the economiser, before exiting the boiler via the stack. The burner tilt is used to change radiation heat distribution between the risers and the superheater.

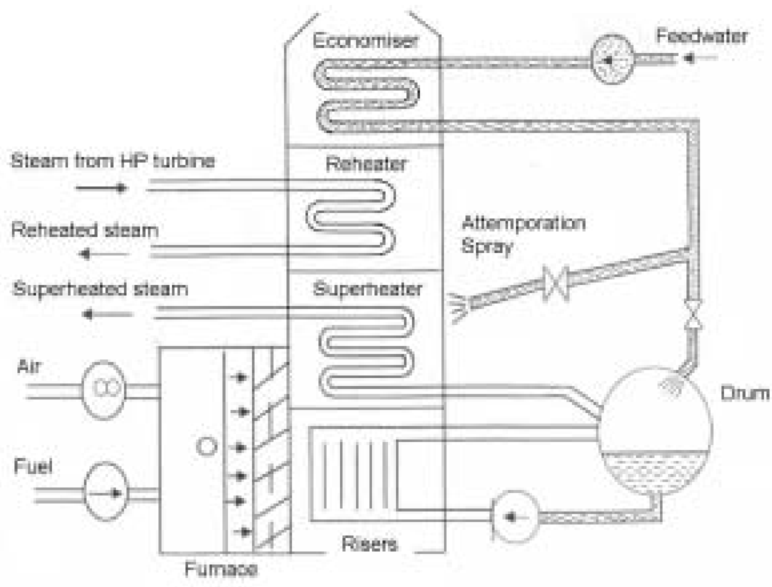



Figure 1: Schematic Diagram of a Boiler

## Model Assumptions

The main assumptions used to construct the mathematical model of a furnace system are as follows [3]:

i. The model only includes time derivatives of variables.
ii. Polynomial fits to steam tables were used to establish the relations between steam parameters such as enthalpy, density, temperature, pressure.
iii. Superheated steam and furnace exhaust gases were treated as ideal gases.

iv. The variables are assumed to satisfy basic physical thermodynamic balances, that is;

Heat balance for a heat exchanger in the form:

$$Q_{in} + w_{in} h_{in} = w_{ou} h_{ou} + V \frac{d}{dt}(\rho_{ou} h_{ou})$$

Mass balance in the form:

$$w_{in} - w_{ou} = \frac{d}{dt}(\rho_{ou} V)$$

Friction losses (momentum balance)

$$P_{in} - P_{ou} = \gamma \frac{w_{in}{}^{2}}{\rho_{in}}$$

v. The heat transfers due to radiation and convection are modeled using:

Stefan-Boltzman law for radiation

$$Q = \frac{K\theta w_g T_g{}^{4}}{\rho_g}$$

Experimental heat transfer equation combustion gas-metal (assuming turbulent combustion gas flow)

$$Q = K w_g{}^{0.6} (T_g - T_m)$$

Experimental heat transfer equation metal-steam (assuming turbulent steam flow)

$$Q = K w_s{}^{0.8} (T_m - T_s)$$

## Mathematical Modeling of Furnace System

The mathematical model of the furnace system is based on first principles physical and thermodynamic laws with appropriate simplifications [3]. The three basic mechanism of heat transfer are conduction, convection and radiation.

Heat balance for combustion

$$C_F w_F + h_A w_A + h_G w_G - Q_{ir} - Q_{is} - w_{EG} R_S \left(1 + \frac{y}{100}\right) \tag{1}$$

$$h_{EG} = V_F \frac{d}{dt}\left(\rho_{EG} h_{EG}\right)$$

Mass balance for combustion

$$w_F + w_A + w_G - w_{EG} = V_F \frac{d}{dt} \rho_{EG} \tag{2}$$

Heat transferred by radiation to risers

$$Q_{ir} = \theta k V_F \sigma T_g^4 \frac{1}{\rho_{EG}} \tag{3}$$

Heat transferred by radiation to the superheater

$$Q_{is} = (1-\theta) k V_F \sigma T_g^4 \frac{1}{\rho_{EG}} \tag{4}$$

Total heat transferred to the superheater

$$Q_{gs} = Q_{is} + k_{gs} w_{EG}^{0.6} (T_{gs} - T_{st}) \tag{5}$$

Heat transferred by convection to the reheater

$$Q_{rs} = k_{rs} w_{EG}^{0.6} (T_{gr} - T_{rh}) \tag{6}$$

Heat transferred by convection to the economiser

$$Q_{es} = k_{es} w_{EG}^{0.6} (T_{ge} - T_{et}) \tag{7}$$

Heat balance for combustion gases flowing across the superheater surface

$$Q_{gs} = w_{EG} c_{gs} (T_g - T_{gr}) + Q_{is} \tag{8}$$

Heat balance for combustion gases flowing across the reheater surface

$$Q_{rs} = w_{EG} c_{gs} (T_{gr} - T_{ge}) \tag{9}$$

Heat balance for combustion gases flowing across the economiser surface

$$Q_{es} = w_{EG} c_{gs} (T_{ge} - T_{gl}) \tag{10}$$

Percentage excess air

$$y = 100(w_A + \gamma_A w_G - w_F R_s) \frac{1}{w_F R_s} \tag{11}$$

Exhaust gas flow through the boiler

$$w_{EG} = k_f p_G \tag{12}$$

Furnace gas pressure

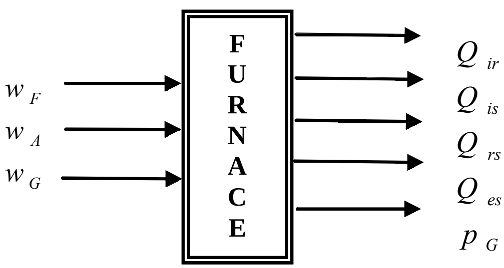
$$p_G = R_{EG} \rho_{EG} T_g \tag{13}$$

Furnace gas temperature

$$T_g = \frac{h_G - h_{ref}}{c_{pg}} + T_{ref} \tag{14}$$

## State Space Modeling for the Furnace System

The state space representation of the furnace system is developed from the mathematical model, which consists of two differential equations, and twelve algebraic equations as stated in the preceding section. Since the state variable in equations (1) and (2) are dependent, the mass balance for combustion in equation (2) is used in the development of the state equation of the furnace system. The three types of variables or parameters used in the development of state space representation for furnace system of combined cycle power plant are the input parameters, output parameters and the state parameter. Based on the mass balance for combustion in equation (2), the state variable is $r_{EG}$ (density of exhaust gas from the boiler in kg/m$^3$). with only three input variables, that is, $w_F$ (fuel flow to the furnace in kg/s), $w_A$ (air flow to the furnace in kg/s), and $w_G$ (exhaust gas flow from the gas turbine in kg/s). This is shown as a simplified block diagram in Figure 2. The rest of the symbols used in modeling the furnace system are listed in Appendix A.




$$x = \frac{d}{dt}\rho_{EG}$$

Figure 2: Simplified Block Diagram of the Furnace System

The state space representation for a given system is not unique but the number of state variables is the same for any of the different state space representation of the same system [9]. The state of a multivariable dynamic system is the smallest set of variables, called state variables, such that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geq t_0$, completely determines the behaviour of the system for any time $t \geq t_0$. Any set of state variables will provide the same information about the systems. Hence, the transformation to a state space representation for the furnace system is discussed in two stages: development of the state equation and then the development of the output equations.

### Development of the State Equation

In the formulation of the state space representation, the state equation can be expressed in a linear form $\dot{x}(t) = A\,x(t) + B\,u(t)$ where $A$ is the state matrix, $B$ is the input matrix, the vector $\dot{x}(t) = dx/dt$ is the time derivatives of state vector $x(t)$ comprising the derivative of each individual element and $u(t)$ is the input vector. Using the mass balance for combustion and equations (12) and (13), we have the following equation.

$$\begin{aligned}\frac{d}{dt}\rho_{EG} &= \frac{1}{V_F}\left(w_F + w_A + w_G - w_{EG}\right)\\ &= \frac{1}{V_F}\left(w_F + w_A + w_G - k_F R_{EG}\rho_{EG}T_g\right)\\ &= -\frac{k_F R_{EG}T_g}{V_F}\rho_{EG} + \frac{w_F + w_A + w_G}{V_F}\end{aligned} \tag{15}$$

The state equation in (15) can be expressed in a matrix-vector form as:

$$\left(\frac{d}{dt}\rho_{EG}\right) = \left(-\frac{k_F R_{EG}T_g}{V_F}\right)\rho_{EG} + \begin{pmatrix}\frac{1}{V_F} & \frac{1}{V_F} & \frac{1}{V_F}\end{pmatrix}\begin{pmatrix}w_F\\ w_A\\ w_G\end{pmatrix} \tag{16}$$

where the state matrix $A = \left(-\dfrac{k_F R_{EG}T_g}{V_F}\right)$

and the input matrix $B = \begin{pmatrix}\frac{1}{V_F} & \frac{1}{V_F} & \frac{1}{V_F}\end{pmatrix}$

### Development of the Output Equations

The output equations can be expressed in a linear form $y(t) = C\,x(t) + D\,u(t)$, where $C$ is the output matrix and $D$ is the direct transmission matrix. The vectors $x(t)$, $u(t)$ and $y(t)$ represent the state, input and output variables respectively. However, in this case we assumed that $D = 0$ which imply that there is no direct transmission between the input $u(t)$ and output $y(t)$. The transformations from the mathematical model to the output equations of the state space model are as follows.

From equation (13),

$$p_G = R_{EG}\rho_{EG}T_g$$

or $$p_G = (R_{EG}T_g)\rho_{EG} \tag{17}$$

From equations (8), (12) and (13),

$$Q_{gs} = w_{EG}c_{gs}(T_g - T_{gr}) + Q_{is}$$

$$Q_{is} = Q_{gs} - w_{EG}c_{gs}(T_g - T_{gr})$$

$$= Q_{gs} - w_{EG}c_{gs}T_g + w_{EG}c_{gs}T_{gr}$$

$$= (Q_{gs} + w_{EG}c_{gs}T_{gr}) - w_{EG}c_{gs}\frac{p_G}{R_{EG}\rho_{EG}}$$

$$R_{EG}\rho_{EG}Q_{is} = (Q_{gs} + w_{EG}c_{gs}T_{gr})R_{EG}\rho_{EG} - w_{EG}c_{gs}p_G$$

$$Q_{is} = (Q_{gs} + w_{EG}c_{gs}T_{gr})R_{EG}\rho_{EG}\frac{T_g}{p_G} - w_{EG}c_{gs}p_G\frac{T_g}{p_G}$$

$$\frac{p_G}{\rho_{EG}T_g}\rho_{EG}Q_{is} = (Q_{gs} + w_{EG}c_{gs}T_{gr})R_{EG}\rho_{EG} - w_{EG}c_{gs}p_G$$

$$Q_{is} = \frac{T_g R_{EG} k_f}{w_{EG}}(Q_{gs} + w_{EG}c_{gs}T_{gr})\rho_{EG} - w_{EG}c_{gs}T_g \tag{18}$$

Dividing equation (3) by equation (4) leads to the following equation.

$$\frac{Q_{ir}}{Q_{is}} = \frac{\theta}{1-\theta}$$

$$Q_{ir} = \left(\frac{\theta}{1-\theta}\right)\left(\frac{T_g R_{EG} k_f}{w_{EG}}(Q_{gs} + w_{EG} c_{gs} T_{gr})\rho_{EG} - w_{EG} c_{gs} T_g\right)$$

$$Q_{ir} = \left(\frac{\theta}{1-\theta}\right)\left(\frac{T_g R_{EG} k_f}{w_{EG}}(Q_{gs} + w_{EG} c_{gs} T_{gr})\rho_{EG}\right) - \left(\frac{\theta}{1-\theta}\right) w_{EG} c_{gs} T_g \tag{19}$$

From equations (9), (12) and (13),

$$Q_{rs} = w_{EG} c_{gs}(T_{gr} - T_{ge})$$

$$Q_{rs} = k_f R_{EG} \rho_{EG} T_g c_{gs}(T_{gr} - T_{ge})$$

or $$Q_{rs} = \left(k_f R_{EG} T_g c_{gs}(T_{gr} - T_{ge})\right)\rho_{EG} \tag{20}$$

In a similar manner, equation (10) is manipulated as

$$Q_{es} = w_{EG} c_{gs}(T_{ge} - T_{gl})$$

$$Q_{es} = k_f R_{EG} \rho_{EG} T_g c_{gs}(T_{ge} - T_{gl})$$

or $$Q_{es} = \left(k_f R_{EG} T_g c_{gs}(T_{ge} - T_{gl})\right)\rho_{EG} \tag{21}$$

Writing equations (18) to (21) in a matrix-vector form, we have the output equations as:

$$\begin{pmatrix} Q_{ir} \\ Q_{is} \\ Q_{rs} \\ Q_{es} \\ p_G \end{pmatrix} = \begin{pmatrix} \frac{\theta T_g R_{EG} k_f}{(1-\theta) w_{EG}}\left(Q_{gs} + w_{EG} c_{gs} T_{gr}\right) \\ \frac{T_g R_{EG} k_f}{w_{EG}}\left(Q_{gs} + w_{EG} c_{gs} T_{gr}\right) \\ k_f R_{EG} T_g c_{gs}(T_{gr} - T_{ge}) \\ k_f R_{EG} T_g c_{gs}(T_{ge} - T_{gl}) \\ R_{EG} T_g \end{pmatrix} \left(\rho_{EG}\right) \tag{22}$$

where the output matrix $$C = \begin{pmatrix} \dfrac{\theta T_g R_{EG} k_f}{(1-\theta) w_{EG}} \left( Q_{gs} + w_{EG} c_{gs} T_{gr} \right) \\ \dfrac{T_g R_{EG} k_f}{w_{EG}} \left( Q_{gs} + w_{EG} c_{gs} T_{gr} \right) \\ k_f R_{EG} T_g c_{gs} (T_{gr} - T_{ge}) \\ k_f R_{EG} T_g c_{gs} (T_{ge} - T_{gl}) \\ R_{EG} T_g \end{pmatrix}$$

Hence, the state space representation of a furnace system is represented by equations (16) and (22), with one state variable, three input parameters and five output parameters. The state and input parameters are mentioned in the preceding section. The five output parameters are $Q_{ir}$ (heat transferred to the risers in J/s), $Q_{is}$ (heat transferred to the superheater in J/s), $Q_{rs}$ (heat transferred to the reheater in J/s), $Q_{es}$ (heat transferred to the economizer in J/s), and $p_G$ (furnace air pressure in Pa).

## The State Space Analysis of the Furnace System

Mathematical software, Matlab® version 6.1, is used to analyse the dynamic behaviour of the state space model of the furnace system, as represented in equations (16) and (22). The steady state operating data [3] is used to analyze its dynamic characteristic. Figure 3 shows the state response $x(t)$ and the output responses $y_1, y_2 \ldots y_5$ of the system resulting from any initial state $x(0)$ with zero input. The resulting responses are computed using the Matlab® m-file programs. These responses describe the evolution with time of the internal and external system variables. The state response will decay to approximately zero in about 11000 time units, while the output responses will decay in about 8000 time units. The accuracy of the approximate value will improve, as the time interval is made smaller. The structural properties that formed the essential requirements in the state space analysis of the furnace system are stability, observability and controllability.

### Stability Analysis

Any dynamic systems respond to an input by undergoing a transient response before reaching a steady state response that generally resembles the input. The furnace system's performance towards a real input signal is tested by using the standard test input, such as the impulse and the step functions. The time responses of the furnace system for step and impulse functions are shown in Figure 4 and Figure 5 respectively (Appendix B). These graphs indicate that the performance of the furnace system towards a real input signal is satisfactory. Intuitively, a system is generally regarded as stable if its response remains finite. The response of linear systems can always be decomposed as the zero-state response and the zero-input response. Lyapunov's First Method [9, 10] is used to evaluate the stability of the zero-input response for the state space representation of the furnace system. It involves an examination of eigenvalues of the system-description matrix *A*. Since the eigenvalues of *A* have negative real parts, the furnace system is asymptotically stable. Since asymptotic stability implies bounded input bounded output stability [10], the furnace system is also bounded input and bounded output stable.

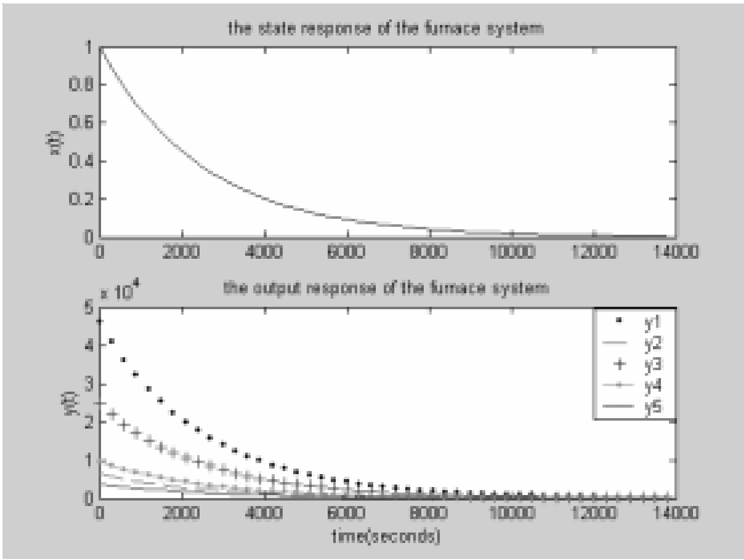



Figure 3: State and Output Responses of the Furnace System

## Controllability and Observability

The dual concepts of controllability and observability are fundamental to the control of multivariable systems, particularly with regard to optimal control. By using the Principle of Duality [9], the observability of a given system can be checked by testing the state controllability of its dual. Complete controllability ensures the existence of an unconstrained control vector, $u(t)$, and thus the existence of a possible controller. On the other hand, complete observability ensures knowledge of the state or internal behaviour, $x(t)$ of the system from knowledge of the output, $y(t)$.. Based on the Algebraic Controllability Theorem [11], the system is controllable if the rank $r(Q)$ of the controllability test matrix $Q = [B\ AB \ldots A^{k-1}\ B]$ is equal to the order of the system. Since $r(Q)$ = order of system = 1, the furnace system is controllable. Similarly, the rank $r(N)$ of the observability test matrix $N = [C' \;\; A'C' \;\; \ldots \;\; (A')^{k-1}C']$ is equal to 1, the order of the system. Thus, the furnace system is controllable and observable.

## Conclusion

In this paper, we have demonstrated a technique of developing a state space model of a furnace system of combined cycle power plant. Our approach is to write the simple derivative equation for each energy-storage element and solve for each derivative term as a linear combination of any of the system variables and the input that are present in the equation. Next, all other system variables in each equation are expressed in terms of the state variables and the input. Finally, we write the output variables as linear combinations of the state variables and the input parameters. The state space analysis of the furnace system are presented to obtain considerable insight of the system behaviour. Although most results that are available for MIMO state space descriptions can now also be obtained in the transfer function approach, the state space formulation stays the most elegant way of dealing with genelizations for MIMO or nonlinear systems. Moreover, in practice the state space formulation is very important for numerical computation and controller design, and can be extended for time-varying systems.

Every system has some uncertainty and constraints associated with it. This uncertainty is characterised and represented in system modeling in order to gain a greater understanding of a system, and therefore aid in the decision-making process. Besides, in most control system the task is to determine the input based on the desired or specified output. Thus, the state space model of a multivariable dynamic system will be extended by considering uncertainties in the parameter. An inverse methodology will be adopted to formulate an algorithm for optimization of input parameters that is implemented to the furnace system.

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## Appendix A

### Nomenclature

| | | | | | |
|---|---|---|---|---|---|
| $C_F$ | - | Fuel calorific value (J/kg) | $T_{rh}$ | - | Reheater metal tube temperature (°K) |
| $c_{pg}$ | - | Specific heat of exhaust gases at constant pressure (Js/kg°K) | $T_{ge}$ | - | Combustion gas temperature at the economise (°K) |
| $c_{gs}$ | - | Combustion gas specific heat capacity (Js/kg°K) | $T_{et}$ | - | Economiser metal tube temperature (°K) |
| $w_F$ | - | Fuel flow (kg/s) | $T_{gl}$ | - | Boiler exhaust gas temperature (°K) |
| $w_A$ | - | Air flow (kg/s) | $T_{ref}$ | - | Reference exhaust gases temperature condition (°K) |
| $w_{EG}$ | - | Gas mass flow through the boiler (kg/s) | $T_m$ | - | Metal temperature (°K) |
| $w_G$ | - | Exhaust gas (from the gas turbine) flow (kg/s) | $T_s$ | - | Steam temperature (°K) |
| $w_{in}$ | - | Inlet mass flow (kg/s) | $R_s$ | - | Stoichiometric air/fuel volume ratio |
| $w_{ou}$ | - | Outlet mass flow (kg/s) | $R_{EG}$ | - | Ideal gas constant for exhaust gases |
| $w_g$ | - | Flow of substances entering combustion (kg/s) | Y | - | The percentage excess air level |
| $h_G$ | - | Exhaust gas (from the gas turbine) specific enthalpy (J/kg) | $V$ | - | Volume ($m^3$) |
| $h_A$ | - | Air specific enthalpy (J/kg) | $V_F$ | - | Combustion chamber volume ($m^3$) |
| $h_{EG}$ | - | Gas specific enthalpy (J/kg) | $\sigma$ | - | Stefan-Boltzman constant |
| $h_{re}$ | - | Reference exhaust gases enthalpy (J/kg) | $\theta$ | - | Burner tilt angle $0 < è < 1$ |
| $h_{in}$ | - | Inlet specific enthalpy (J/kg) | $\gamma_A$ | - | Content of fresh air in exhaust from gas turbine |
| $h_{ou}$ | - | Outlet specific enthalpy (J/kg) | ã | - | Friction coefficient |
| $Q_{ir}$ | - | Heat transferred by radiation to risers (J/s) | $\rho_{EG}$ | - | Gas density ($kg/m^3$) |
| $Q_{is}$ | - | Heat transferred by radiation to the superheater (J/s) | ñ | - | Specific density ($kg/m^3$) |
| $Q_{rs}$ | - | Heat transferred to reheater (J/s) | $p_G$ | - | Gas pressure in furnace (Pa) |
| $Q_{gs}$ | - | Heat transferred to superheater (J/s) | $p_{in}$ | - | Inlet pressure (Pa) |
| $Q_{es}$ | - | Heat transferred to the economizer (J/s) | $p_{ou}$ | - | Outlet pressure (Pa) |
| $Q_{in}$ | - | Incoming heat flow (kg/s) | $k$ | - | Attenuation coefficient |
| $Q$ | - | Heat flow (J/s) | $k_{gs}$ | - | An experimental coefficient (J/kg°K) |
| $T_g$ | - | Gas temperature (°K) | $k_{rs}$ | - | An experimental coefficient (J/kg°K) |
| $T_{gs}$ | - | Gas temperature at the superheater (°K) | $k_{es}$ | - | An experimental coefficient (J/kg°K) |
| $T_{st}$ | - | Superheater metal tube temperature (°K) | $k_f$ | - | A friction coefficient |
| $T_{gr}$ | - | Combustion gas temperature at the reheater (°K) | $K$ | - | A coefficient |

## Appendix B

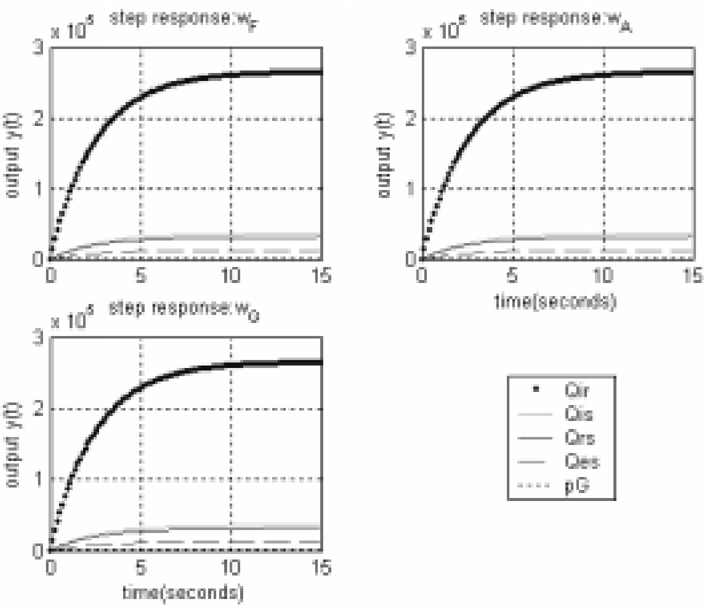


Figure 4: Furnace System Subjected to Step Function

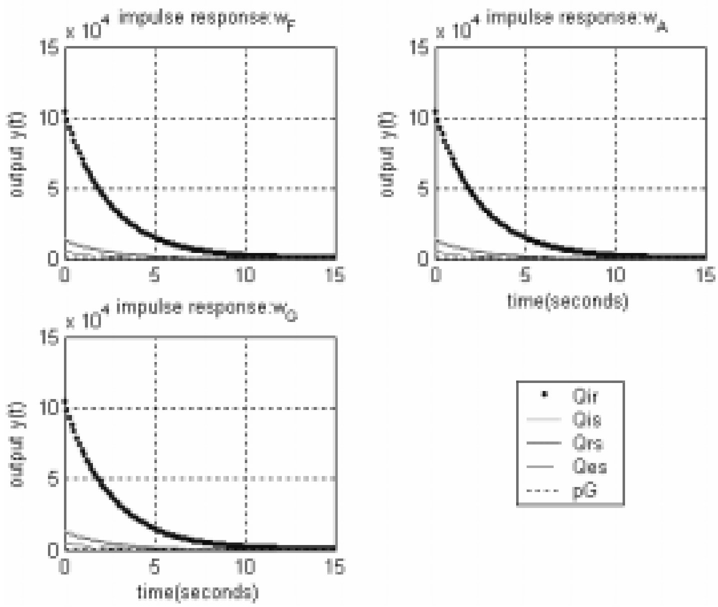


Figure 5: Furnace System Subjected to Impulse Function